

Wave Mechanics of Risk

*A New Paradigm
for Actuarial Science*

Foundations, Theory, and Applications to
Catastrophe Risk Pricing and Insurance

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Preface

This book was born from years of practical experience, intellectual curiosity, and the privilege of working alongside remarkable professionals.

It draws inspiration from my years of consulting work for COFACE, where questions of credit risk, uncertainty, and resilience were not abstract concepts but daily realities. It was further nourished by the many stimulating exchanges with the Sesame team led by Bruno Vial, whose depth of vision and technical rigour constantly challenged conventional ways of thinking. Equally formative were the strategic board discussions held in the presence of Jérôme Cazes and François Meunier, where the broader economic, institutional, and human dimensions of risk became impossible to ignore.

These experiences revealed a simple but powerful truth: the mathematical tools we use to price, reserve, and manage risk were largely designed for a world of thin tails, linear dependencies, and static exposures. Yet insurance exists precisely to confront a more complex reality — one shaped by contagion, structural breaks, feedback effects, and rare events that redefine entire systems.

The question that drove this work was deceptively simple: *What if risk is not a random variable, but a wave?*

This question led us, perhaps inevitably, to Louis de Broglie's revolutionary 1924 insight — that matter itself has a wave nature — and to Erwin Schrödinger's 1926 operationalization of that insight into an equation that transformed physics. Our thesis is that an analogous operationalization is both possible and necessary in actuarial science.

The reader will find here a rigorous but accessible development of what we call *Wave Mechanics of Risk*: a theoretical framework in which every risk carries a wave function, risks interfere and superpose, catastrophes propagate as shock waves, and the imaginary unit i is not a mathematical curiosity but the fundamental carrier of phase information between correlated risks.

We have written this book for the actuarial scientist who is not afraid of differential equations, for the mathematical physicist curious about applications to insurance, and for the risk manager who suspects that something essential is being missed by standard models. No prior knowledge of quantum mechanics is

assumed, though comfort with probability theory and stochastic processes will be beneficial.

The journey from Gaussian distributions to wave functions is long. We begin, as one must, with a thorough critique of where we currently stand.

Abdelkader Bousabaa

Paris, August 2026

Wave Mechanics of Risk

A NEW PARADIGM FOR ACTUARIAL SCIENCE

The standard tools of actuarial science — Gaussian distributions, linear correlations, static exposure models — were designed for a world that does not exist. Insurance exists precisely to confront a far more complex reality: contagion, structural breaks, feedback effects, and rare events that redefine entire systems.

This book proposes a fundamental paradigm shift. Drawing on the wave-particle duality of Louis de Broglie and the wave equation of Erwin Schrödinger, it develops a complete theoretical framework in which every risk carries a wave function. Risks interfere and superpose; catastrophes propagate as shock waves; the uncertainty principle governs the fundamental limits of actuarial precision; and quantum tunnelling models the reinsurance of tail events that classical theory deems impossible.

Topics covered

De Broglie–Schrödinger formalism • Wave function of risk • Risk interference & superposition • Actuarial uncertainty principle • Energy-level pricing • Wave-based IBNR reserves • Reinsurance & quantum tunnelling • Portfolio decoherence • Catastrophe shock waves • Cat Nat pricing • Climate change drift

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Researcher and educator specialising in financial markets, catastrophe risk modelling, and the application of theoretical physics to actuarial science.