

*Volume II*

# Wave Mechanics of Risk

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*A New Paradigm  
for Actuarial Science*

Numerical Recipes

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# Contents

|                                                                   |           |
|-------------------------------------------------------------------|-----------|
| <b>Preface</b>                                                    | <b>xv</b> |
| <b>Introduction: How to Read This Volume</b>                      | <b>1</b>  |
| <b>I The Theory of Risk Waves</b>                                 | <b>7</b>  |
| <b>1 De Broglie, Schrödinger, and the Birth of Wave Formalism</b> | <b>9</b>  |
| 1.1 The Algebra of Plane Waves . . . . .                          | 10        |
| 1.1.1 Plane Waves in Complex Polar Form . . . . .                 | 10        |
| 1.1.2 Superposition and the Interference Formula . . . . .        | 11        |
| 1.2 The De Broglie Relation . . . . .                             | 14        |
| 1.2.1 Phase as a Lorentz Scalar . . . . .                         | 14        |
| 1.2.2 The De Broglie Relation . . . . .                           | 15        |
| 1.2.3 Worked Example: Four Wavelengths . . . . .                  | 16        |
| 1.3 The Schrödinger Equation . . . . .                            | 17        |
| 1.3.1 Heuristic Derivation . . . . .                              | 17        |
| 1.3.2 Structure of the Hamiltonian . . . . .                      | 19        |
| 1.3.3 Stationary States . . . . .                                 | 20        |
| 1.3.4 Hermiticity and Conservation of the Norm . . . . .          | 20        |
| 1.3.5 The Born Rule and the Probability Current . . . . .         | 22        |
| 1.4 The Particle in a Box . . . . .                               | 23        |
| 1.4.1 The Eigenvalue Problem . . . . .                            | 24        |
| 1.4.2 The Spectrum . . . . .                                      | 24        |
| 1.4.3 Numerical Recipe: Finite Differences . . . . .              | 25        |
| 1.4.4 Worked Example: Electron in a One-Angström Box . . . . .    | 26        |
| 1.5 Three Theorems on the Imaginary Unit . . . . .                | 27        |
| 1.5.1 Oscillation . . . . .                                       | 28        |
| 1.5.2 Reality of the Probability Current . . . . .                | 28        |
| 1.5.3 Interference . . . . .                                      | 28        |
| Chapter Summary . . . . .                                         | 29        |

|          |                                                                               |           |
|----------|-------------------------------------------------------------------------------|-----------|
| <b>2</b> | <b>The Risk Wave Function and the Risk Hamiltonian</b>                        | <b>31</b> |
| 2.1      | The Risk Wave Function . . . . .                                              | 32        |
| 2.1.1    | Setting . . . . .                                                             | 32        |
| 2.1.2    | Definition and the Actuarial Born Rule . . . . .                              | 32        |
| 2.1.3    | The Classical Density as a Real-Valued Risk Wave Function . . . . .           | 34        |
| 2.2      | Three Canonical Risk Wave Functions . . . . .                                 | 34        |
| 2.2.1    | Polar Form . . . . .                                                          | 35        |
| 2.2.2    | Travelling-Wave Ansatz . . . . .                                              | 35        |
| 2.2.3    | Gaussian Wave Packet . . . . .                                                | 36        |
| 2.2.4    | Worked Example: The Coastal Storm Wave Packet . . . . .                       | 38        |
| 2.3      | The Risk Potential . . . . .                                                  | 39        |
| 2.3.1    | Definition . . . . .                                                          | 39        |
| 2.3.2    | Three Canonical Profiles . . . . .                                            | 40        |
| 2.3.3    | Discretisation on a Finite Grid . . . . .                                     | 41        |
| 2.4      | The Risk Hamiltonian and the Actuarial Schrödinger Equation . . . . .         | 42        |
| 2.4.1    | Definition . . . . .                                                          | 42        |
| 2.4.2    | The Actuarial Schrödinger Equation . . . . .                                  | 42        |
| 2.4.3    | Hermiticity, Conservation, and the Probability Current . . . . .              | 43        |
| 2.4.4    | Dispersive Gaussian Wave Packet . . . . .                                     | 43        |
| 2.5      | The Actuarial Quantum of Action $\hbar_r$ . . . . .                           | 46        |
| 2.5.1    | Dimensional Analysis . . . . .                                                | 46        |
| 2.5.2    | Calibration via the Interference Residual . . . . .                           | 47        |
| 2.5.3    | Numerical Recipe: Method of Moments on the Interference<br>Residual . . . . . | 49        |
| 2.6      | Stationary States and Risk Energy Levels . . . . .                            | 50        |
| 2.6.1    | The Time-Independent Actuarial Schrödinger Equation . . . . .                 | 50        |
| 2.6.2    | Spectral Type for the Three Canonical Potentials . . . . .                    | 51        |
| 2.6.3    | Worked Example: A Two-Well Portfolio . . . . .                                | 52        |
| 2.7      | Risk Measures as Expectation Values . . . . .                                 | 53        |
| 2.7.1    | The Loss Operator and Expected Loss . . . . .                                 | 53        |
| 2.7.2    | Value at Risk and Tail Value at Risk . . . . .                                | 54        |
| 2.7.3    | The Pure Premium Formula . . . . .                                            | 55        |
| 2.7.4    | Numerical Recipe . . . . .                                                    | 56        |
| 2.8      | A Complete Example: Coastal Storm Risk . . . . .                              | 57        |
| 2.8.1    | Specification . . . . .                                                       | 57        |
| 2.8.2    | Stationary Spectrum . . . . .                                                 | 57        |
| 2.8.3    | Time Propagation . . . . .                                                    | 58        |
| 2.8.4    | Conditional Density Ratio: Town A vs. Town B . . . . .                        | 58        |
| 2.8.5    | Comparison: Rigid Parametric vs. Dispersive Packet . . . . .                  | 59        |
| 2.8.6    | Convergence Study . . . . .                                                   | 60        |
|          | Chapter Summary . . . . .                                                     | 60        |

|          |                                                                            |           |
|----------|----------------------------------------------------------------------------|-----------|
| <b>3</b> | <b>Interference, Superposition, and the Entanglement of Risks</b>          | <b>63</b> |
| 3.1      | The Mathematical Framework . . . . .                                       | 64        |
| 3.1.1    | Single-Risk and Joint Spaces . . . . .                                     | 64        |
| 3.1.2    | Product States and the Preview of Entanglement . . . . .                   | 65        |
| 3.2      | The Superposition Principle for Risk Wave Functions . . . . .              | 65        |
| 3.2.1    | Closure of $\mathcal{H}_1$ under Superposition . . . . .                   | 66        |
| 3.2.2    | The Interference Formula . . . . .                                         | 66        |
| 3.2.3    | When Does the Interference Vanish? . . . . .                               | 68        |
| 3.2.4    | Numerical Recipe: Computing the Interference Field . . . . .               | 69        |
| 3.3      | The Four Canonical Interference Scenarios . . . . .                        | 70        |
| 3.3.1    | Constructive Interference: $\Delta\phi = 0$ . . . . .                      | 70        |
| 3.3.2    | Destructive Interference: $\Delta\phi = \pi$ . . . . .                     | 71        |
| 3.3.3    | Partial Interference: $0 <  \Delta\phi  < \pi$ . . . . .                   | 71        |
| 3.3.4    | The Classical Limit: $\Delta\phi$ Uniformly Distributed . . . . .          | 72        |
| 3.3.5    | Worked Example: Numerical Demonstration of the Four<br>Scenarios . . . . . | 72        |
| 3.4      | Superposition of $N$ Risk Waves and the Phasor Picture . . . . .           | 73        |
| 3.4.1    | The General $N$ -Wave Superposition . . . . .                              | 73        |
| 3.4.2    | The Phasor Picture . . . . .                                               | 75        |
| 3.4.3    | Numerical Recipe: $N$ -Wave Loss Density on a Grid . . . . .               | 76        |
| 3.4.4    | Worked Example: A Ten-Peril Typhoon . . . . .                              | 77        |
| 3.5      | Standing Waves and Spatial Interference Patterns . . . . .                 | 78        |
| 3.5.1    | The Standing Wave from Counter-Propagating Waves . . . . .                 | 78        |
| 3.5.2    | Unequal Wavelengths . . . . .                                              | 79        |
| 3.5.3    | Worked Example: French Atlantic Coast Accumulation<br>Zones . . . . .      | 80        |
| 3.6      | Decoherence: The Decoherence Parameter . . . . .                           | 81        |
| 3.6.1    | Definition and Elementary Bounds . . . . .                                 | 81        |
| 3.6.2    | The Decoherence Bound on the Interference Correction . . . . .             | 82        |
| 3.6.3    | Three Conventional Regimes . . . . .                                       | 83        |
| 3.6.4    | Numerical Recipe . . . . .                                                 | 83        |
| 3.7      | Risk Entanglement . . . . .                                                | 84        |
| 3.7.1    | The Schmidt Decomposition . . . . .                                        | 85        |
| 3.7.2    | Reduced Density Matrix and Entanglement Entropy . . . . .                  | 86        |
| 3.7.3    | Numerical Recipe: Schmidt Decomposition by SVD . . . . .                   | 87        |
| 3.7.4    | Three Canonical Actuarial Examples . . . . .                               | 88        |
| 3.7.5    | Implications for Economic Capital . . . . .                                | 89        |
|          | Chapter Summary . . . . .                                                  | 90        |
| <b>4</b> | <b>The Actuarial Uncertainty Principle</b>                                 | <b>93</b> |
| 4.1      | The Robertson–Schrödinger Inequality . . . . .                             | 94        |
| 4.1.1    | Standard Deviation of a Hermitian Operator . . . . .                       | 94        |
| 4.1.2    | The Inequality . . . . .                                                   | 95        |
| 4.1.3    | The Heisenberg Position–Momentum Bound . . . . .                           | 96        |
| 4.1.4    | The Energy–Time Bound . . . . .                                            | 96        |

|       |                                                             |     |
|-------|-------------------------------------------------------------|-----|
| 4.2   | The Fourier Uncertainty . . . . .                           | 97  |
| 4.2.1 | The Fourier Transform on $L^2$ . . . . .                    | 97  |
| 4.2.2 | The Fourier Uncertainty Inequality . . . . .                | 98  |
| 4.3   | Conjugate Variables in Actuarial Science . . . . .          | 99  |
| 4.3.1 | The Severity Operator . . . . .                             | 99  |
| 4.3.2 | The Frequency Operator . . . . .                            | 99  |
| 4.3.3 | The Commutator and the Actuarial Bound . . . . .            | 100 |
| 4.3.4 | Units and Dimensional Consistency . . . . .                 | 100 |
| 4.4   | The Detective and the Storm: A Worked Calculation . . . . . | 102 |
| 4.4.1 | Synthetic Event Archive . . . . .                           | 102 |
| 4.4.2 | Two Estimation Strategies . . . . .                         | 102 |
| 4.4.3 | Numerical Result . . . . .                                  | 103 |
| 4.5   | Three Manifestations in Actuarial Practice . . . . .        | 104 |
| 4.5.1 | The Return-Period Paradox . . . . .                         | 104 |
| 4.5.2 | The GLM Frequency–Severity Dichotomy . . . . .              | 105 |
| 4.5.3 | The Tail-Uncertainty Question . . . . .                     | 106 |
| 4.6   | The Coherent Risk: Minimum-Uncertainty State . . . . .      | 106 |
| 4.6.1 | The Gaussian Saturates the Bound . . . . .                  | 107 |
| 4.6.2 | Squeezing . . . . .                                         | 107 |
| 4.6.3 | Three Squeezed States in Practice . . . . .                 | 108 |
| 4.7   | Implications for Regulatory Capital . . . . .               | 109 |
| 4.7.1 | The SCR Estimation Floor: An Open Conjecture . . . . .      | 109 |
| 4.7.2 | The Minimum-Uncertainty Prudence Loading . . . . .          | 110 |
| 4.8   | The Uncertainty Principle and Model Risk . . . . .          | 110 |
|       | Chapter Summary . . . . .                                   | 112 |

## II Actuarial Applications 115

|          |                                                               |            |
|----------|---------------------------------------------------------------|------------|
| <b>5</b> | <b>Pricing by Quantised Energy Levels</b>                     | <b>117</b> |
| 5.1      | The Quantum Harmonic Oscillator . . . . .                     | 118        |
| 5.1.1    | Setup . . . . .                                               | 118        |
| 5.1.2    | Ladder Operators . . . . .                                    | 118        |
| 5.1.3    | The Spectrum . . . . .                                        | 119        |
| 5.2      | The Pure Premium from Energy Levels . . . . .                 | 121        |
| 5.2.1    | Premium of the $n$ -th Energy Level . . . . .                 | 121        |
| 5.2.2    | Rate on Line as a Discrete Sum . . . . .                      | 122        |
| 5.3      | Three Canonical Spectra . . . . .                             | 123        |
| 5.3.1    | The Three Theorems . . . . .                                  | 123        |
| 5.3.2    | Comparison Table . . . . .                                    | 124        |
| 5.3.3    | Worked Example: Three Spectra of a Single Portfolio . . . . . | 124        |
| 5.4      | Transitions and the Franck–Condon Factor . . . . .            | 125        |
| 5.4.1    | The Transition Matrix Element . . . . .                       | 125        |
| 5.4.2    | The Franck–Condon Factor . . . . .                            | 126        |
| 5.4.3    | Worked Example: A Climate-Driven Frequency Shift . . . . .    | 127        |

|          |                                                                  |            |
|----------|------------------------------------------------------------------|------------|
| 5.5      | The Energy-Level Tariff . . . . .                                | 128        |
| 5.5.1    | Definition . . . . .                                             | 128        |
| 5.5.2    | Stability of the Energy-Level Tariff . . . . .                   | 129        |
| 5.5.3    | The Minimum Meaningful Rate Increment . . . . .                  | 130        |
| 5.6      | Selection Rules . . . . .                                        | 130        |
| 5.6.1    | The $\Delta n = \pm 1$ Rule . . . . .                            | 130        |
| 5.6.2    | Higher-Order Selection Rules . . . . .                           | 131        |
| 5.7      | Worked Example: French Flood Insurance . . . . .                 | 131        |
| 5.7.1    | Calibration . . . . .                                            | 131        |
| 5.7.2    | Analytical Spectrum . . . . .                                    | 132        |
| 5.7.3    | Numerical Spectrum from the Finite-Difference Recipe . . . . .   | 132        |
| 5.7.4    | Level Premium Rates . . . . .                                    | 133        |
| 5.7.5    | Comparison with the Flat CatNat Rate . . . . .                   | 134        |
| 5.7.6    | The Reform Programme . . . . .                                   | 134        |
|          | Chapter Summary . . . . .                                        | 135        |
| <b>6</b> | <b>Provisioning by Wave Equation</b>                             | <b>137</b> |
| 6.1      | The IBNR Wave Function and the Development Hamiltonian . . . . . | 138        |
| 6.1.1    | The Development Space . . . . .                                  | 138        |
| 6.1.2    | The IBNR Wave Function . . . . .                                 | 138        |
| 6.1.3    | The Development Hamiltonian . . . . .                            | 139        |
| 6.2      | The Development Wave Equation . . . . .                          | 139        |
| 6.2.1    | The Equation . . . . .                                           | 140        |
| 6.2.2    | The Development Probability Current . . . . .                    | 140        |
| 6.3      | Stationary Development Modes . . . . .                           | 141        |
| 6.3.1    | Separation and the Time-Independent Equation . . . . .           | 141        |
| 6.3.2    | Harmonic Development Modes . . . . .                             | 142        |
| 6.3.3    | Modes as Development Archetypes . . . . .                        | 142        |
| 6.3.4    | Numerical Recipe: Modal Decomposition of a Triangle . . . . .    | 144        |
| 6.4      | The Chain-Ladder as a Single-Mode Approximation . . . . .        | 144        |
| 6.4.1    | The Age-to-Age Factor as a Ratio of Mode Amplitudes . . . . .    | 145        |
| 6.4.2    | The Systematic Chain-Ladder Error . . . . .                      | 146        |
| 6.5      | Long-Tail Lines and the Tail Factor . . . . .                    | 147        |
| 6.5.1    | Ground-Mode Dominance . . . . .                                  | 147        |
| 6.5.2    | The Tail Factor Formula . . . . .                                | 147        |
| 6.5.3    | Numerical Recipe: Estimating $\omega_r^{(0)}$ . . . . .          | 148        |
| 6.6      | Non-Stationarity: Inflation and Legal Change . . . . .           | 149        |
| 6.6.1    | Inflation: Amplitude Drift and Frequency Drift . . . . .         | 149        |
| 6.6.2    | Legal Change as a Potential Step . . . . .                       | 150        |
| 6.6.3    | Diagnostic for Non-Stationarity . . . . .                        | 151        |
| 6.7      | Worked Example: UK Employers' Liability . . . . .                | 151        |
| 6.7.1    | Calibration . . . . .                                            | 152        |
| 6.7.2    | Tail Factor . . . . .                                            | 152        |
| 6.7.3    | Reserve Comparison . . . . .                                     | 153        |
| 6.7.4    | Convergence Study . . . . .                                      | 153        |

|                                                                     |            |
|---------------------------------------------------------------------|------------|
| Chapter Summary . . . . .                                           | 154        |
| <b>7 Reinsurance and Quantum Tunnelling . . . . .</b>               | <b>157</b> |
| 7.1 The Rectangular Potential Barrier . . . . .                     | 158        |
| 7.1.1 The Three Regions . . . . .                                   | 158        |
| 7.1.2 Wave Function in the Three Regions . . . . .                  | 159        |
| 7.1.3 Matching Conditions . . . . .                                 | 160        |
| 7.2 The Transmission Coefficient . . . . .                          | 160        |
| 7.2.1 Exact Formula . . . . .                                       | 160        |
| 7.2.2 Thick-Barrier Asymptotic . . . . .                            | 161        |
| 7.2.3 Numerical Recipe: Discrete Transmission via FD . . . . .      | 163        |
| 7.3 Three Actuarial Manifestations . . . . .                        | 163        |
| 7.3.1 Sub-Attachment Penetration . . . . .                          | 163        |
| 7.3.2 Above-Limit Penetration . . . . .                             | 164        |
| 7.3.3 Reinstatement Premium Distortion . . . . .                    | 165        |
| 7.4 The Actuarial Tunnel Formula and Its Sensitivities . . . . .    | 165        |
| 7.4.1 The Tunnel Formula Restated . . . . .                         | 166        |
| 7.4.2 Three Sensitivity Theorems . . . . .                          | 166        |
| 7.4.3 Numerical Recipe . . . . .                                    | 167        |
| 7.5 The WKB Approximation . . . . .                                 | 167        |
| 7.5.1 The Eikonal Ansatz . . . . .                                  | 168        |
| 7.5.2 The WKB Transmission Formula . . . . .                        | 168        |
| 7.5.3 Reduction to the Rectangular Case . . . . .                   | 169        |
| 7.5.4 Numerical Recipe . . . . .                                    | 170        |
| 7.6 Resonance: When the Layer Becomes Transparent . . . . .         | 170        |
| 7.6.1 Above-Barrier Transmission . . . . .                          | 170        |
| 7.6.2 Resonance Condition . . . . .                                 | 171        |
| 7.6.3 The Sub-Barrier Anti-Resonance . . . . .                      | 172        |
| 7.6.4 Actuarial Resonance-Avoidance Design . . . . .                | 172        |
| 7.7 Worked Example: French Windstorm Cat XL . . . . .               | 173        |
| 7.7.1 Programme Specification . . . . .                             | 173        |
| 7.7.2 Classical First-Layer Pricing . . . . .                       | 173        |
| 7.7.3 Energy Levels and Sub-Attachment Tunnelling . . . . .         | 174        |
| 7.7.4 Over-Barrier Severities: Partial Reflection . . . . .         | 174        |
| 7.7.5 Resonance Check . . . . .                                     | 175        |
| 7.7.6 Complete Wave-Mechanical Premium . . . . .                    | 175        |
| Chapter Summary . . . . .                                           | 176        |
| <b>8 Portfolio Decoherence and Risk Management . . . . .</b>        | <b>179</b> |
| 8.1 The Density Matrix and Decoherence . . . . .                    | 180        |
| 8.1.1 Density Matrix of a Pure State . . . . .                      | 180        |
| 8.1.2 Density Matrix of a Mixed State . . . . .                     | 180        |
| 8.1.3 Decoherence as Suppression of Off-Diagonal Elements . . . . . | 181        |
| 8.1.4 The Lindblad Master Equation . . . . .                        | 181        |
| 8.2 The Decoherence Time . . . . .                                  | 182        |

|       |                                                              |     |
|-------|--------------------------------------------------------------|-----|
| 8.2.1 | The General Scaling . . . . .                                | 182 |
| 8.2.2 | The Actuarial Coherence Time . . . . .                       | 183 |
| 8.3   | The Decoherence Spectrum . . . . .                           | 183 |
| 8.3.1 | Four Archetypes . . . . .                                    | 183 |
| 8.3.2 | Modelling Implications . . . . .                             | 184 |
| 8.4   | The Coherence Matrix and the Decoherence Parameter . . . . . | 185 |
| 8.4.1 | Portfolio Density Matrix . . . . .                           | 185 |
| 8.4.2 | The Decoherence Parameter . . . . .                          | 185 |
| 8.4.3 | Bounds and Extremes . . . . .                                | 186 |
| 8.4.4 | Empirical Estimation . . . . .                               | 186 |
| 8.5   | Decoherence as a Management Tool . . . . .                   | 187 |
| 8.5.1 | The Optimisation Problem . . . . .                           | 187 |
| 8.5.2 | Wave-Mechanical Diversification . . . . .                    | 188 |
| 8.6   | The Solvency II Standard Formula . . . . .                   | 189 |
| 8.6.1 | The Standard Formula as a Coherence Matrix . . . . .         | 189 |
| 8.6.2 | The Correction Theorem . . . . .                             | 190 |
| 8.6.3 | Internal Model Validation . . . . .                          | 191 |
| 8.7   | Dynamic Decoherence . . . . .                                | 191 |
| 8.7.1 | The Rate Equation . . . . .                                  | 191 |
| 8.7.2 | Climate Change as a Coherence Engine . . . . .               | 192 |
| 8.7.3 | Numerical Recipe: Integration of the Rate Equation . . . . . | 193 |
| 8.8   | Worked Example: French Multi-Line Insurer Audit . . . . .    | 193 |
| 8.8.1 | Portfolio Composition . . . . .                              | 194 |
| 8.8.2 | Sub-Portfolio Decoherence Parameters . . . . .               | 194 |
| 8.8.3 | Aggregate Wave-Mechanical SCR . . . . .                      | 195 |
|       | Chapter Summary . . . . .                                    | 196 |

### III Natural Catastrophe Risk: A Deep Dive 197

#### 9 Natural Catastrophes as Shock Waves 199

|       |                                                       |     |
|-------|-------------------------------------------------------|-----|
| 9.1   | The Four Physical Wave Equations . . . . .            | 200 |
| 9.1.1 | Seismic Waves . . . . .                               | 200 |
| 9.1.2 | Cyclonic Wind Waves . . . . .                         | 201 |
| 9.1.3 | Kinematic Flood Waves . . . . .                       | 201 |
| 9.1.4 | Wildfire Reaction–Diffusion Waves . . . . .           | 202 |
| 9.2   | Construction of the Risk Wave Function . . . . .      | 202 |
| 9.2.1 | The HVE Construction . . . . .                        | 202 |
| 9.2.2 | The Loss Density . . . . .                            | 203 |
| 9.2.3 | Total Event Loss and Its Temporal Emergence . . . . . | 204 |
| 9.3   | The Speed of Loss Propagation . . . . .               | 204 |
| 9.3.1 | The Temporal-Sequence Theorem . . . . .               | 204 |
| 9.3.2 | Three Consequences for Claims Operations . . . . .    | 205 |
| 9.3.3 | Worked Example: Marne Flood Propagation . . . . .     | 206 |
| 9.4   | Shock Waves and the Mach Cone . . . . .               | 206 |

|           |                                                                                |            |
|-----------|--------------------------------------------------------------------------------|------------|
| 9.4.1     | The Mach Cone . . . . .                                                        | 206        |
| 9.4.2     | The PML as a Shock-Wave Maximum . . . . .                                      | 207        |
| 9.4.3     | Worked Example: Wildfire Under Storm-Force Winds . . . . .                     | 209        |
| 9.5       | Multi-Peril Interference: Compound Events . . . . .                            | 209        |
| 9.5.1     | The Compound-Event Loss Density . . . . .                                      | 209        |
| 9.5.2     | Worked Example: Hurricane Harvey 2017 . . . . .                                | 211        |
| 9.6       | The Cat Nat Risk Hamiltonian . . . . .                                         | 211        |
| 9.6.1     | Peril-Specific Kinetic Operators . . . . .                                     | 212        |
| 9.6.2     | Unified Hamiltonian Framework . . . . .                                        | 213        |
| 9.7       | The Propagation Speed as the Missing Parameter . . . . .                       | 214        |
| 9.7.1     | Three Information Channels Carried by $c$ . . . . .                            | 214        |
| 9.7.2     | The Linking Proposition . . . . .                                              | 214        |
| 9.7.3     | Estimating $c$ from Historical Events . . . . .                                | 215        |
|           | Chapter Summary . . . . .                                                      | 216        |
| <b>10</b> | <b>Wave-Mechanical Pricing of Cat Nat Premiums</b>                             | <b>219</b> |
| 10.1      | The Wave-Mechanical Premium Formula . . . . .                                  | 220        |
| 10.1.1    | Single-Peril Premium as a Conditional Expectation . . . . .                    | 220        |
| 10.1.2    | Multi-Peril Premium: Additive Expectation, Interfering<br>Volatility . . . . . | 221        |
| 10.1.3    | Operator Form . . . . .                                                        | 222        |
| 10.2      | Calibration Methodology . . . . .                                              | 222        |
| 10.2.1    | Step 1: The Event Frequencies . . . . .                                        | 223        |
| 10.2.2    | Step 2: The Cat Nat Wave Functions . . . . .                                   | 223        |
| 10.2.3    | Step 3: The Risk Load from Co-Timing . . . . .                                 | 224        |
| 10.3      | Case Study 1: French Atlantic Windstorm . . . . .                              | 224        |
| 10.3.1    | Calibration Inputs . . . . .                                                   | 225        |
| 10.3.2    | Per-Location Wave Function . . . . .                                           | 226        |
| 10.3.3    | Premium Map . . . . .                                                          | 226        |
| 10.3.4    | Convergence Study . . . . .                                                    | 227        |
| 10.4      | Case Study 2: Italian Earthquake . . . . .                                     | 227        |
| 10.4.1    | The Seismic Risk Wave Function . . . . .                                       | 227        |
| 10.4.2    | Site Resonance Theorem . . . . .                                               | 228        |
| 10.4.3    | Worked Example: Umbria–Marche Portfolio . . . . .                              | 229        |
| 10.5      | Case Study 3: Mediterranean Drought . . . . .                                  | 230        |
| 10.5.1    | Drought as a Diffusion Wave . . . . .                                          | 230        |
| 10.5.2    | The Long Correlation Length . . . . .                                          | 231        |
| 10.5.3    | Worked Example: The European Drought of 2022 . . . . .                         | 232        |
| 10.6      | The Universal Premium Surface . . . . .                                        | 233        |
| 10.6.1    | Three Regimes of Spatial Structure . . . . .                                   | 233        |
| 10.6.2    | Sharp- versus Smooth-Gradient Surfaces . . . . .                               | 233        |
| 10.6.3    | Multi-Peril Composition and Interference Signatures . . . . .                  | 234        |
| 10.7      | Pricing the Uncertainty: The Prudence Loading . . . . .                        | 234        |
| 10.7.1    | The Prudence Loading . . . . .                                                 | 234        |
| 10.7.2    | Risk-Aversion Calibration . . . . .                                            | 235        |

|           |                                                                              |            |
|-----------|------------------------------------------------------------------------------|------------|
| 10.7.3    | Worked Example: Prudence Loading for the French Atlantic Portfolio . . . . . | 235        |
|           | Chapter Summary . . . . .                                                    | 236        |
| <b>11</b> | <b>Accumulation Zones and Territorial Interference</b>                       | <b>239</b> |
| 11.1      | Standing Waves in One and Two Dimensions . . . . .                           | 240        |
| 11.1.1    | One-Dimensional Recap . . . . .                                              | 240        |
| 11.1.2    | Two-Dimensional Generalisation . . . . .                                     | 241        |
| 11.1.3    | Antinode Locations . . . . .                                                 | 242        |
| 11.2      | The Territorial Interference Map . . . . .                                   | 242        |
| 11.3      | The Fresnel Zone . . . . .                                                   | 243        |
| 11.3.1    | The Fresnel Zone Radius . . . . .                                            | 244        |
| 11.3.2    | Higher Fresnel Zones . . . . .                                               | 245        |
| 11.4      | Case Study A: The North Sea Coastal Zone . . . . .                           | 246        |
| 11.4.1    | Calibration . . . . .                                                        | 246        |
| 11.4.2    | Antinode Spacing . . . . .                                                   | 246        |
| 11.4.3    | Validation Status: An Open Protocol . . . . .                                | 247        |
| 11.5      | Case Study B: The Rhône Valley . . . . .                                     | 247        |
| 11.5.1    | Calibration . . . . .                                                        | 248        |
| 11.5.2    | Three-Peril Constructive Maximum . . . . .                                   | 248        |
| 11.5.3    | Worked Example: The Gard Department . . . . .                                | 248        |
| 11.6      | Case Study C: The Italian Po Plain . . . . .                                 | 249        |
| 11.6.1    | Calibration . . . . .                                                        | 249        |
| 11.6.2    | Long-Wavelength Limit . . . . .                                              | 250        |
| 11.7      | Managing Accumulation Zones . . . . .                                        | 251        |
| 11.7.1    | Response 1: Premium Loading . . . . .                                        | 251        |
| 11.7.2    | Response 2: Aggregate-Exposure Cap . . . . .                                 | 251        |
| 11.7.3    | Response 3: Reinsurance Ring-Fencing . . . . .                               | 252        |
| 11.7.4    | Response 4: Geographic Withdrawal . . . . .                                  | 252        |
| 11.8      | Dynamic Accumulation . . . . .                                               | 253        |
| 11.8.1    | Antinode-Drift Rate Equation . . . . .                                       | 253        |
| 11.8.2    | Worked Example: Climate-Driven Antinode Shift . . . . .                      | 253        |
|           | Chapter Summary . . . . .                                                    | 254        |
| <b>12</b> | <b>Climate Change and the Drift of Wave Parameters</b>                       | <b>257</b> |
| 12.1      | Climate Change as a Drift in the Hamiltonian . . . . .                       | 258        |
| 12.1.1    | The Perturbed Hamiltonian . . . . .                                          | 258        |
| 12.1.2    | The Four Drifting Parameters . . . . .                                       | 258        |
| 12.1.3    | Empirical Calibration of the Drift Rates . . . . .                           | 259        |
| 12.2      | The Adiabatic Theorem . . . . .                                              | 260        |
| 12.2.1    | The Adiabatic Theorem . . . . .                                              | 260        |
| 12.2.2    | The Actuarial Adiabatic Condition . . . . .                                  | 261        |
| 12.2.3    | Three Climate Regimes . . . . .                                              | 261        |
| 12.3      | Prospective Premium Trajectories . . . . .                                   | 262        |
| 12.3.1    | The Drift-Adjusted Premium . . . . .                                         | 262        |

|                                             |                                                                                  |            |
|---------------------------------------------|----------------------------------------------------------------------------------|------------|
| 12.3.2                                      | Worked Example: European Windstorm Under RCP Scenarios . . . . .                 | 263        |
| 12.3.3                                      | Adiabaticity Check . . . . .                                                     | 264        |
| 12.4                                        | The Phase Drift: When Accumulation Zones Move . . . . .                          | 264        |
| 12.4.1                                      | Multi-Parameter Antinode-Drift Theorem . . . . .                                 | 264        |
| 12.4.2                                      | Numerical Recipe . . . . .                                                       | 265        |
| 12.4.3                                      | Worked Example: Drift of the Dutch Antinodes . . . . .                           | 265        |
| 12.5                                        | The Coherence Length Explosion . . . . .                                         | 266        |
| 12.5.1                                      | The Coherence Length Theorem . . . . .                                           | 266        |
| 12.5.2                                      | Threshold Crossing . . . . .                                                     | 267        |
| 12.5.3                                      | Worked Example: A Coastal Portfolio Crossing the Threshold . . . . .             | 269        |
| 12.6                                        | The Non-Stationarity Test . . . . .                                              | 270        |
| 12.6.1                                      | The Likelihood-Ratio Framework . . . . .                                         | 270        |
| 12.6.2                                      | Rolling-Window Spectral Tracker . . . . .                                        | 271        |
| 12.7                                        | Building the Climate-Adaptive Wave Model . . . . .                               | 272        |
| 12.7.1                                      | The Three-Layer Architecture . . . . .                                           | 272        |
| 12.7.2                                      | Operational Cadence . . . . .                                                    | 272        |
| 12.7.3                                      | Adiabaticity-Conditional Adaptation . . . . .                                    | 272        |
| 12.8                                        | Climate Bonds and the Quantum of Carbon Risk . . . . .                           | 273        |
| 12.8.1                                      | Wave-Mechanical Climate Bonds . . . . .                                          | 273        |
| 12.8.2                                      | The Quantum of Carbon Risk . . . . .                                             | 274        |
| 12.8.3                                      | Worked Example: Pricing a Climate Bond on European Windstorm Intensity . . . . . | 274        |
|                                             | Chapter Summary . . . . .                                                        | 275        |
| <b>Conclusion: From Recipes to Practice</b> |                                                                                  | <b>277</b> |
|                                             | Frontier 1. Genomic Mortality . . . . .                                          | 277        |
|                                             | Frontier 2. Ecological Risk . . . . .                                            | 278        |
|                                             | Frontier 3. Systemic Financial Risk . . . . .                                    | 278        |
|                                             | Frontier 4. Molecular Insurance . . . . .                                        | 278        |
|                                             | Frontier 5. Sovereign Risk . . . . .                                             | 279        |
|                                             | Frontier 6. Pandemic Wave Function . . . . .                                     | 279        |
|                                             | Frontier 7. Algorithmic Risk . . . . .                                           | 280        |
|                                             | Frontier 8. Post-Quantum Cryptographic Risk . . . . .                            | 280        |
|                                             | Frontier 9. Energy Transition . . . . .                                          | 280        |
|                                             | Frontier 10. Space and Orbital Debris . . . . .                                  | 281        |
|                                             | Frontier 11. Longevity and Ageing . . . . .                                      | 281        |
|                                             | Frontier 12. Biodiversity Collapse . . . . .                                     | 281        |
|                                             | A Computational Research Agenda . . . . .                                        | 282        |
|                                             | Software Roadmap and Reproducibility . . . . .                                   | 282        |
|                                             | The Final Word . . . . .                                                         | 283        |
| <b>References</b>                           |                                                                                  | <b>285</b> |

**Index**

**291**



# Preface

The first volume of *Wave Mechanics of Risk* laid out a paradigm: risk is a wave, the catastrophe a shock front, the portfolio a superposition. The argument was deliberately conceptual — a book of intuitions and ideas.

This second volume is its mirror image. Where the first volume argued in prose, this one computes. Where it gestured at a derivation, this one carries it out. Every chapter here bears the same number and title as a chapter in the first volume, and exists to give the reader the mathematics, the statistics, and the fully worked numerical examples that stand behind the intuition.

The reader is assumed to be familiar with the first volume, or to read the two volumes side by side. Each chapter opens with a brief cross-reference to its counterpart and then proceeds rigorously: definitions, derivations, recipes, and worked examples.

The printed book is, by design, only half of the project. The companion website `quantum.abdelkader.site` hosts the Python implementations of every numerical recipe, the datasets used in every worked example, and an interactive application through which a large share of the mathematics and the exercises can be explored, modified, and executed. No source code appears in these pages: the book gives the algorithm in mathematical form, and the website gives the algorithm in running form. The two are meant to be used together.

The spirit is that of *Numerical Recipes*: a working companion the practitioner keeps on the desk, not on the shelf.

*Abdelkader Bousabaa*  
Paris, August 2026

# Wave Mechanics of Risk

NUMERICAL RECIPES · VOLUME II

The first volume of *Wave Mechanics of Risk* laid out a paradigm: risk as a wave, the catastrophe as a shock front, the portfolio as a superposition. This second volume is its mirror image: where the first argued in prose, this one computes.

Chapter by chapter — in strict correspondence with the first volume — it supplies the definitions, the derivations, the numerical recipes and the fully worked examples that stand behind the intuition: the actuarial Schrödinger equation and its spectrum, interference and entanglement of risks, the uncertainty relation, energy-level pricing, wave-based reserving, barrier penetration and reinsurance, portfolio decoherence, and the catastrophe chapters brought to computational form.

The printed book is half of the project: the companion website hosts the Python implementation of every recipe, the datasets of every worked example, and an interactive application through which the mathematics can be explored, modified, and run.

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## Topics covered

Actuarial Schrödinger equation • Born rule & risk measures • Interference, entanglement & Schmidt decomposition • The actuarial uncertainty relation • Energy-level pricing • Wave-based reserving • Barrier penetration & reinsurance • Portfolio decoherence & the SCR • Cat Nat shock waves, pricing & accumulation • Climate drift • Numerical recipes & worked examples

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