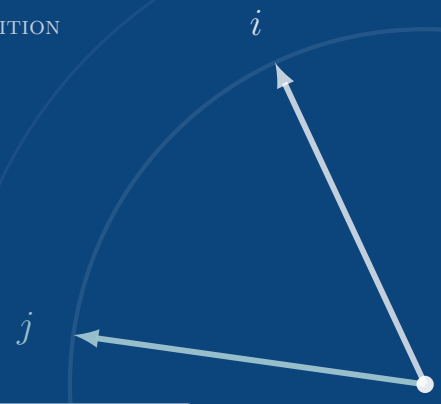
# The $i$ and the $j$

*One Century of Experiments on the Foundations of Quantum Mechanics*

From Planck's Quantum to the Quaternionic Question, 1900–2026

**Abdelkader BOUSABAA**

Second Edition




Seven Lions Editions

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# Preface

This book tells one story. It is the story of a number.

In 1925, a strange guest entered the equations of physics. It was the imaginary number $i$, the square root of $-1$. Schrödinger did not invite it. He resisted it. But the equation refused to work without it. A century later, the guest is still there. Every laser, every atomic clock, every quantum computer obeys an equation built on $i$.

This fact hides a question. Nature could have chosen simpler numbers. She could also have chosen richer ones. There exists a larger family of numbers, the quaternions, where a second imaginary unit $j$ lives next to $i$. The question is simple to state. Does nature use $i$ alone, or is there a small amount of $j$ in the world?

This question is not philosophy. It is measurable. Physicists have interrogated it with neutrons, with photons, with kaons, with entangled pairs, with atomic clocks. Each experiment is a beautiful story, and each one closed a door with a number. This book narrates these experiments in the order of history, from Planck in 1900 to the complete map of 2026.

A word on how the book was born, because its method is older than its subject. Years before the program, the author directed research at Rothschild, in London. The central instrument of that office was a whiteboard. Everything that crossed it, markets, risks, models, had to survive one test. Could it be explained, in one minute, to a non specialist, with one simple drawing? The conviction forged on that board, and tested on it a thousand times, was that everything can, quantum physics included, and that whoever cannot draw an idea does not yet possess it. In those London years, quantum physics served as the reservoir of parallels, waves, interferences and superpositions lending their images to the movements of markets. One day the direction of service reversed. The parallels deserved to become the subject.

The subject found its instrument in 2026, when a research program of numerical benches was constructed to reread the great records of modern physics, Bell tests, atomic clocks, entangled processors, against the question of the quaternions. The program produced a map, the one that occupies the final Part. It also produced a realization. Behind that map stood a century of experiments that

form one continuous story, and that story had never been told from its beginning to its end for the general reader. This book is that story. It inherits from the program its bookkeeping, thresholds written in advance, sources recomputed, errors printed rather than erased. It inherits from the century its heroes, most of them machines. And it inherits from the whiteboard of London its method, visible on every page. One drawing per machine. One parallel per idea. One minute of explanation, honestly attempted, for every notion in the story.

## For whom this book is written

Three readers accompanied the writing of every page.

The first is the curious reader, with no physics at all. For this reader the book carries its parallels from daily life, one per idea, its original drawings, one per machine, and its refusal of jargon. The story can be read as a story, from the first page to the last, without opening a single box.

The second is the student or the engineer. For this reader the boxes exist, the formulas are exact, the six block structure makes every chapter a self contained file, and the glossary and timeline of the appendices serve as instruments of revision.

The third is the researcher. For this reader the final Part presents a complete program with its laws, its audits and its two costed proposals, the appendices carry the recipes for recomputing the central numbers, and the bibliography gives one line of honest guidance per source. The reader in a hurry can also enter by the pilot, Chapter 14, one complete experiment told in full, and decide there whether this book speaks their language.

## The promise of this book

You do not need any prior knowledge. The text explains every notion at its first apparition. The mathematics never blocks the road. When a calculation merits attention, it lives in a separate box. You may ignore every box and still understand the story.

Each experiment is presented in six fixed blocks. The epoch situates the moment. The question states what was at stake, in one sentence. The apparatus shows the machine, with an original drawing. The intuition offers a parallel from daily life. The measured number gives the true result. The final block explains what changed, what closed, and what opened.

## The rule of honesty

This book does not announce a discovery. The $j$ has never been observed. Every experiment in this book produced a ceiling, never a signal. The value of the story is elsewhere. It resides in the method, in the beauty of the machines, and in a surprising map. Many instruments of modern physics test the $j$ without knowing it. The reader will discover them one by one.

Every number in this book is traceable. It comes from a published article or from a numerical bench that the reader can execute at home. The final appendix explains how.

The book itself was treated as an experiment. Every external number was recomputed from published tables where tables exist. Every bench figure regenerates identically from a fixed seed. The bibliography was verified against every claim of the text. And when the drafting process caught its own errors, a bound attributed to the wrong year, a curve displaced from its points, the corrections were applied and, where instructive, printed in place, because a book that preaches verification must accept its own medicine.

## Why this is a book and not a course

One last word on what this volume was meant to be, since its shape carries the trace of the change.

It began as a set of lecture notes. The plan was a teaching document for students, the kind of duplicated text that circulates in a department, with the six block structure serving a syllabus and the appendices serving a problem sheet. Much of what the reader holds was written under that intention, which explains the insistence on explaining every notion at its first apparition and the refusal to assume anything beyond the complex numbers.

The intention changed while the writing continued. The author decided to undertake a doctorate, and a doctorate consumes the years that a course of this size demands. The choice was therefore between leaving the manuscript unfinished on a disk and finishing it as something other than a course.

The second was chosen, and the manuscript became a book for the general reader. The conversion cost less than expected, and the reason is worth one sentence for anybody in the same position. A teaching text that has genuinely refused jargon, and that explains each idea with a parallel from daily life rather than with a prerequisite, is already most of the way to a book for everybody. What had to be added was the story, the people and the machines. What had to be removed was nothing at all.

If one reader, somewhere, closes this book and looks differently at the first symbol of one equation, the book has done its work.

Abdelkader Bousabaa,
Paris, August 2026.

*Nature had three arithmetics.*

*She chose the middle one.*
*How do we know?*

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In 1925 the imaginary number $i$ entered the equations of physics. Schrödinger resisted it. The equation refused to work without it. A century later, every laser, every atomic clock and every quantum computer obeys that equation. Behind this success hides a question. Mathematics permits a richer arithmetic, the quaternions, where a second unit $j$ lives beside $i$. Does nature use a small amount of it?

This book tells the century that asked the question properly, from Planck's grain of energy in 1900 to the complete map of 2026. Neutron interferometers, kaon beams, loophole-free Bell tests, clocks of eighteen digits, entangled processors. Each experiment is told in six fixed blocks, with one intuition per idea, one drawing per machine, and the true numbers. No prior knowledge is required. Every number is traceable to a published table, or to a bench the reader can run at home.

The $j$ has never been found. What was found is a map of four sectors, a ceiling in each, and two doors designed for the next decade.

Abdelkader BOUSABAA directed research at Rothschild in London and is a researcher at the University of Paris-Saclay.