

Financial Wave Mechanics

*A Quantum Approach to
Pricing and Risk Management*

From De Broglie Waves to Option Pricing,
Hedging, and Financial Risk

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Preface

This book is the product of thirty years of thinking about finance, mathematics, and the invisible threads that connect them. It did not emerge from a single moment of insight but from a long accumulation of questions, conversations, experiments, and — more than anything — a stubborn refusal to accept that the most important phenomena in financial markets are “anomalies” that defy explanation.

The story begins in the late 1990s, at the *Crédit Lyonnais*. I was a young quantitative analyst, still shaping my understanding of how markets actually work, and I had the extraordinary good fortune to join the *Groupe de Recherche Opérationnelle* (GRO) — a small team of mathematicians and economists embedded within the bank, tasked with bringing rigour to problems that the trading floor solved by intuition and experience. It was there that I completed my real formation, the one that no university can provide: the formation that comes from watching theory meet reality, every day, in the unforgiving laboratory of the markets.

The GRO was a remarkable place, and I owe an immeasurable debt to the people who made it so. Thierry Roncalli, whose intellectual breadth and generosity shaped my understanding of quantitative risk management, and with whom I shared, and still share, countless Friday evening dinners where the conversation ranged from copula theory to philosophy to the meaning of probability itself. Antoine Frachot, whose clarity of thought and rigour of method set a standard I have aspired to ever since. Alain Charmant, whose practical wisdom and deep knowledge of credit risk taught me that the most elegant mathematics is worthless if it cannot be explained to a trader in five minutes. These men were not just colleagues; they were the companions of my intellectual formation, and the ideas in this book carry the echo of those early conversations, even if the conversations themselves have long since ended.

It was during those years at the *Crédit Lyonnais* that I first noticed the resemblance between the Black–Scholes equation and the Schrödinger equation. The observation was not original — it had been made by others before me, and it was widely regarded as a mathematical curiosity with no practical significance. But something about it troubled me. In physics, when two equations have the same structure, it usually means that the underlying phenomena share a common mechanism. The diffusion equation and the heat equation describe fundamentally similar processes (the random spreading of particles, the random spreading of thermal energy),

and this structural similarity is not a coincidence but a consequence of shared physics. Could the same be true of the Black–Scholes equation and the Schrödinger equation? Could the mathematics of quantum mechanics be telling us something real about how markets work?

I did not have the tools to answer that question in the 1990s. I filed it away and moved on.

The next chapter of this story unfolds in the classrooms of the French universities and *grandes écoles* where I have taught for twenty-seven years. Teaching is the most honest form of learning: you cannot hide behind jargon when a bright student asks “but *why*?” and will not accept “because the model says so” as an answer. Over nearly three decades of lecturing on derivatives pricing, risk management, stochastic calculus, and financial engineering, I was forced to confront the same questions, year after year, that the classical framework answers poorly: Why does the volatility smile exist? Why do barriers breach more often than the model predicts? Why do defaults cluster? Why do correlations spike in a crisis? Why does VaR systematically underestimate tail risk?

Each of these questions has an accepted answer within the classical framework, but each answer is, in its own way, a patch: the smile is “explained” by stochastic volatility (an additional diffusion bolted onto the price process), the barrier premium is “explained” by discrete monitoring adjustments, the default clustering is “explained” by a Gaussian copula (a correlation structure borrowed from statistics), the VaR failure is “explained” by fat-tailed distributions (an alternative to the Gaussian that is chosen precisely because it fits the data better). None of these explanations is wrong, exactly, but none is *structural*: none tells you *why* the phenomenon occurs, only *how to model it once you have observed it*.

It was the students, with their relentless questions, who kept the old observation alive in my mind. If the Black–Scholes equation is the Schrödinger equation without the i , then what happens if you put the i back? What phenomena appear? Could they be the phenomena that the classical framework cannot explain?

The third strand of this story is my years as a consultant in Paris, working with banks, asset managers, and insurance companies on the practical problems of derivatives pricing, model validation, and risk management. Consulting teaches you what academia does not: that a model is only as good as its ability to survive contact with messy data, impatient traders, skeptical regulators, and the relentless pressure of daily P&L. The ideas in this book have been shaped by that pressure. Every formula in these pages was developed with a practical question in mind: Can this be calibrated from market data? Can it be computed in real time? Does it improve the hedge? Does it pass the backtest? A model that is mathematically beautiful but practically useless belongs in a physics journal, not in a finance book.

The consulting years also taught me something about communication. The most powerful idea is worthless if it cannot be explained to a non-specialist. I have worked with brilliant traders who never studied quantum mechanics, with risk managers who care about tail risk but not about wave functions, and with regulators who want to understand the model risk of any new approach before they approve it. This book is written for all of them. I have worked hard to make the ideas intuitive and didactic — to use analogies, examples, and visualisations that make the quantum framework accessible to anyone with a quantitative finance background, even if they have never heard of de Broglie or Schrödinger.

The final strand is more recent: years spent building and validating quantitative models in production environments, where the gap between what the textbooks promise and what the markets deliver is felt most acutely. It is this gap — between the elegance of the classical framework and the stubbornness of the empirical anomalies — that ultimately convinced me that a structural alternative was needed, not another patch.

The result of these four currents — the Crédit Lyonnais formation, the decades of teaching, the Parisian consulting years, and the years in production — is the book you hold in your hands. I have tried to make it three things at once: rigorous (every result is derived, not asserted), intuitive (every formula is accompanied by an analogy or a visualisation), and practical (every chapter includes worked examples with realistic numbers). Whether I have succeeded is for the reader to judge.

I owe thanks to many more people than I can name. To the students across twenty-seven years of lectures who asked the questions that shaped my thinking. To the traders, risk managers, and regulators who taught me what matters in practice. To the colleagues in academia and industry who discussed these ideas, challenged them, and helped me refine them. And to the pioneers of econophysics — Jean-Philippe Bouchaud, Marc Potters, Rama Cont, and many others — who showed, decades before this book, that the tools of physics have a legitimate place in the study of financial markets.

Any errors that remain are mine alone. I offer this book not as a finished theory but as a beginning — a first exploration of a territory that I believe is rich with undiscovered structure. The wave is launched. I invite the reader to help determine where it propagates.

Abdelkader Bousabaa
Paris, August 2026

Put the imaginary unit back into Black–Scholes, and the smile, the barrier premium and the default cluster stop being anomalies.

The Black–Scholes equation is the Schrödinger equation without its i . Restoring it turns option pricing into wave mechanics: two competing market signals interfere and produce the volatility smile; asset values tunnel through solvency barriers; correlations lock together under stress. Every result is derived in closed form, calibrated on real S&P 500 data, and worked through with numbers.

Volume I builds the framework from first principles and applies it across three asset classes and the full risk management stack: vanilla and exotic options with quantum Greeks, interest rates and negative yields as tunnelling, credit default and correlation as entanglement, then Value at Risk, stress testing, model risk, liquidity and systemic risk. Calibrated states of the S&P 500 index anchor the numbers throughout. The companion **Volume II**, *Numerical Recipes for Financial Wave Mechanics*, supplies the algorithms, calibration procedures and production tools.

Abdelkader Bousabaa is a quantitative finance practitioner and lecturer based in Paris. Trained at Crédit Lyonnais, he has taught quantitative finance for more than twenty-five years and worked as a consultant for banks, asset managers and insurers on derivatives pricing, model validation and risk management. He founded and developed an asset management company in London, and has built and validated quantitative models in production.